Solve the following System

$$\begin{cases} w + x + y + z = 3 \\ -w + x - 2y + z = -2 \\ 2w + 2x - y + z = 3 \\ 3w + x + y - z = 5 \end{cases}$$

variables	Elimination
2	1
3	3
4	6
5	10
6	15
n	$\frac{n(n-1)}{2}$
10	45
77	≈ 2900

warm up: $w+x+y+z=3$

$-w+x-2y+z=-2$

$2w+2x-y+z=3$

$3w+x+y-z=5$

$(2x-y+2z=1)-2$

$\underline{4x-5y+3z=-1}$

$-4x+2y-4z=-2$

$-3y-z=-3$

$-3-z=-3$

$+3 \qquad +3$

$-z=0$

$\boxed{z=0}$

$w+x+y+z=3$

$-w+x-2y+z=-2$

$\underline{2x-y+2z=1}$

$-2w+2x-4y+2z=-4$

$2w+2x-y+z=3$

$\underline{4x-5y+3z=-1}$

$-2w-2x-2y-z=-6$

$2w+2x-y+z=3$

$\underline{-3y=-3}$

$\boxed{y=1}$

$2x-1=1$

$+1 \quad +1$

$2x=2$

$\boxed{x=1}$

$w+1+1=3$

$\boxed{w=1}$

$1+1+1=3$

$3=3$ ✓

$$3x - y + z = 3$$
$$x + y + 2z = 4$$
$$x + 2y + z = 4$$

$$3x - \cancel{y} + z = 3$$
$$x + \cancel{y} + 2z = 4$$
$$\boxed{4x + 3z = 7}$$

$$(2)\ 3x - y + z = 3$$
$$x + 2y + z = 4$$

$$6x - \cancel{2y} + 2z = 6$$
$$x + \cancel{2y} + z = 4$$
$$\boxed{7x + 3z = 10}$$

$$4x + \cancel{3z} = 7$$
$$-\ 7x + \cancel{3z} = 10$$
$$-3x = -3$$
$$\boxed{x = 1}$$

$$4x + 3z = 7$$
$$4(1) + 3z = 7$$
$$\boxed{z = 1}$$

$$x + y + 2z = 4$$
$$1 + y + 2(1) = 4$$
$$\boxed{y = 1}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$$

$$A \cdot X = B$$

$$\frac{\cancel{A}X}{\cancel{A}} = \frac{B}{A}$$

$$X = \frac{B}{A}$$

$$\underline{\underline{X = A^{-1} \cdot B}}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{2 \cdot 1 - 1 \cdot 1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}}_{A^{-1}} \cdot \underbrace{\begin{bmatrix} 8 \\ 10 \end{bmatrix}}_{B} = \begin{bmatrix} 2 \cdot 8 + -1 \cdot 10 \\ -1 \cdot 8 + 1 \cdot 10 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

Solve using calculator

$$x + 2y = 10$$
$$3x + 5y = 26$$

$$X = A^{-1} \cdot B$$